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Quantum corrections for the Reisner-Nordström charged black hole

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ABSTRACT

We calculate the quantum corrections of geometric and thermodynamic quantities for the Reisner-Nordström charged black hole, within the context of 2D spherically symmetric dilaton gravity model. Special attention is payed to the quantum corrections of the extreme Reisner-Nordström solution. We find a state of the extreme black hole with regular behaviour at the horizon.

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1 Introduction

Many aspects of the radiation of black holes were investigated since its discovery in 1975 [1]. The work which has been done in the last years is mainly done in the framework of various dilaton two dimensional (2D) models, due to renewed interest in 2D gravity originated in the string theory. In this context, two main types of models have been analyzed. One of them are string-inspired models with black-hole solutions (CGHS, RST, BPP) which are exactly solvable. The motivation to analyze the other class of 2D models like null-dust model, spherically symmetric gravity (SSG), etc., is that they are obtained from 4D gravity action by some variant of dimensional reduction and it is expected that they can reproduce some aspects of 4D gravitational phenomena.

In this paper we analyze the quantum corrections to the Reisner-Nordström black hole in the framework of SSG. Extreme black holes have an important place in some recent investigations. First, as extreme black hole is the solution with critical behaviour [2], i.e. a sort of phase transition, it obeys some scaling laws. This enables one to discuss such questions as problem of information loss, avoiding the limits which the quanization of gravity (issues of the existence of Planck length) puts. Second, extreme dilaton black holes are representations of massive single string states in superstring theories while extreme black p-branes occur as solutions in M-theory; therefore it is of importance to analyze the properties of these and similar solutions.

One of the extreme black hole solutions is the Reisner-Nordström charged black hole. There are two main streams of thought about the thermodynamic properties of this solution. Hawking [3] argues that, as this solution has a double zero on the horizon and therefore does not have a conical singularity (in its Euclidean exstension), one can prescribe it arbitrary temperature. This means that extreme black hole can be in equilibrium with its radiation at arbitrary temperature. Furthermore, this property implies that the entropy of extreme black hole is zero, i.e. it is not equal to the fourth of its area as it would be according to the Bekenstein-Hawking formula. This point of view was further reinforced by the discussion of Teitelboim [4], which partially also implies that the extreme solution is stable, as it has different topology from nonextreme solutions. On the other hand, the calculations of Trivedi [5] and Anderson, Hiscock and Loranz [6, 7] show that the extreme black hole at nonzero temperature cannot exist (is not stable), because the energy-momentum tensor (EMT) of the radiation diverges on the horizon in the semiclassical approximation. There is a further discussion about stability at zero temperature: Trivedi obtained that, in his particular 2D model, EMT diverges for zero temperature on the horizon also, but ”mildly”, in such a way that the curvature and the tidal forces remain finite. The calculations of [6, 7] suggest that in 4D and $T = 0$, EMT is finite on the horizon. So, as it can be seen, the questions of temperature and entropy of extreme Reisner-Nordström black hole are still unsettled.

In our previous paper [8], we calculated the backreaction of the radiation on the Schwarzschild solution in the framework of the SSG dilaton gravity model. The effective action which was used to describe the one-loop quantum effects was found

in [9, 10, 11, 12]. In the SSG model the coupling of the scalar field to gravity is supposed to be more realistic (from the point of view of four dimensions), because it is obtained from the 4D minimal coupling of scalar field to gravity. This is new in comparison to the previous calculations [5], and it is therefore interesting to see whether it will produce also some differencies in the conclusions about the behaviour of the Reisner-Nordström solution. Also, our approach for finding the value of EMT and identification of Hartle-Hawking vacuum proves to be relatively simple in the Schwarzchild case, which is a further motivation to see what can this model tell about the behaviour of the extreme solution. Indeed, an unexpected result comes out: within this model there is a Hartle-Hawking vacuum with fixed (possibly nonzero) temperature, defined in the sense that the semiclassical value of the EMT tensor is regular everywhere (except at the physical singularity $r = 0$). Unfortunately, the temperature of this state cannot be determined, as the scalar field is not free in the asymptotic region and therefore the Stefan-Boltzmann law cannot be applied. The question of stability of this state is also an interesting point to consider.

The plan of this paper is the following: In the second section we define the model and obtain the quantum corrections for the nonextreme Reisner-Nordström solution. In the third section we discuss properties of the extreme Reisner-Nordström solution. The analysis of the possibility to define the Hartle-Hawking vacuum and the Kruscal coordinates is also given. A generalization of our model to the model with ξRf^2 coupling and some results are given in the Appendix.

2 The Model and its Solution

The starting point of our consideration is the 4D action

$$\Gamma_0 = \frac{1}{16\pi G}\int d^4x\sqrt{-g^{(4)}}(R^{(4)} - F^2) - \frac{1}{8\pi}\sum_i \int d^4x\sqrt{-g^{(4)}}((\nabla f_i)^2 + \xi R f_i^2) \ , \quad (1)$$

which describes the Einstein gravity coupled with electromagnetic field A_μ and scalar fields f_i ($i = 1, \ldots N$). The $\xi R f_i^2$ interaction term is added to the action. Specially, if $\xi = \frac{1}{6}$, the second term in (1) is conformally invariant. Using the equation of motion for electromagnetic field and the solution which corresponds to the Coulomb potential (as it is done in [13]) and performing the spherically symmetric reduction, from action (1) we get 2D action

$$\begin{aligned}\Gamma_0 &= \frac{1}{4G}\int d^2x\sqrt{-g}\Big(e^{-2\Phi}(R + 2(\nabla\Phi)^2 + 2e^{2\Phi}(1 - Q^2e^{2\Phi})) \\ &- 2Ge^{-2\Phi}\sum_i((\nabla f_i)^2 + \xi f_i^2(R - 6(\nabla\Phi)^2 + 4\Box\Phi + 2e^{2\Phi}))\Big) \ , \qquad (2)\end{aligned}$$

where Φ is the dilaton field and Q is the electric charge. When we add the one-loop quantum correction for the matter fields f_i to the action (2), we get the nonlocal

effective action. It is given by (see Appendix):

$$
\begin{aligned}
\Gamma_1 &= \frac{1}{4G}\int d^2x\sqrt{-g}\Big(e^{-2\Phi}(R+2(\nabla\Phi)^2+2e^{2\Phi}(1-Q^2e^{2\Phi})) \\
&- 2Ge^{-2\Phi}\sum_i((\nabla f_i)^2+\xi f_i^2(R-6(\nabla\Phi)^2+4\Box\Phi+2e^{2\Phi}))\Big) \\
&- \frac{N}{8\pi}\int d^2x\sqrt{-g}\Big((\frac{1}{12}-\xi)R\frac{1}{\Box}R+(1-4\xi)R\Phi \\
&+ (-1+6\xi)R\frac{1}{\Box}(\nabla\Phi)^2-2\xi R\frac{1}{\Box}e^{2\Phi}\Big)\ . \qquad (3)
\end{aligned}
$$

The effective action (3) can be rewritten in the local form by introducing two auxillary fields, ψ and χ ($\xi \neq \frac{1}{12},\ \frac{1}{6}$)

$$
\begin{aligned}
\Gamma_1 &= \frac{1}{4G}\Big[\int d^2x\sqrt{-g}\Big(e^{-2\Phi}(R+2(\nabla\Phi)^2+2e^{2\Phi}(1-Q^2e^{2\Phi})) \\
&- 2Ge^{-2\Phi}\sum_i(\nabla f_i)^2+\xi f_i^2(R-6(\nabla\Phi)^2+4\Box\Phi+2e^{2\Phi}))\Big) \\
&- \kappa(1-12\xi)\int d^2x\sqrt{-g}\Big((\nabla\psi)^2+2R\psi+b(\nabla\psi)(\nabla\chi) \\
&+ b\psi(\nabla\Phi)^2+bR\chi+c\psi e^{2\Phi}+dR\Phi\Big)\Big]\ . \qquad (4)
\end{aligned}
$$

In (4) the following notation is introduced: $\kappa = \frac{GN}{24\pi}$, $b = -12\frac{1-6\xi}{1-12\xi}$, $c = \frac{-24\xi}{1-12\xi}$, $d = 12\frac{1-4\xi}{1-12\xi}$. The auxilliary fields ψ, χ obey the equations of motion:

$$\Box\psi = R \qquad (5)$$

$$\Box\chi = (\nabla\Phi)^2+\frac{c}{b}e^{2\Phi} \qquad (6)$$

Substitution of (5) and (6) into (4) gives the nonlocal action (3). In the case $\xi = \frac{1}{12}$, the local form of the one-loop correction is given by

$$\Gamma^{(1)} = \frac{N}{48\pi}\int d^2x\sqrt{-g}\Big(R\chi+(3(\nabla\Phi)^2+e^{2\Phi})\psi+\nabla\psi\nabla\chi-4R\Phi\Big)\ , \qquad (7)$$

where the auxilliary fields obey $\Box\psi = R$, and $\Box\chi = 3(\nabla\Phi)^2+e^{2\Phi}$. In the other special case $\xi = \frac{1}{6}$ we get

$$\Gamma^{(1)} = \frac{N}{48\pi}\int d^2x\sqrt{-g}\Big(R\psi+2R\chi+2\psi e^{2\Phi}+\frac{1}{2}(\nabla\psi)^2+2\nabla\psi\nabla\chi-2R\Phi\Big)\ , \qquad (8)$$

where $\Box\psi = R$, $\Box\chi = e^{2\Phi}$.

The action (3) in the classical limit $\kappa = 0$ posesses the Reisner-Nordström black hole as a vacuum solution. Since the scalar fields f_i enter the action (3) quadratically, we can get the desired quantum correction to the vacuum solution by introducing $f_i = 0$ directly to the action and then obtaining the equations of motion. Furthermore,

we will take also $\xi = 0$ for simplicity, and discuss the case $\xi \neq 0$ briefly in the Appendix. The action which we will vary is

$$\begin{aligned}\Gamma_1 &= \frac{1}{4G}\int d^2x\sqrt{-g}\Big(r^2R+2(\nabla r)^2+2U(r)\Big)\\ &- \frac{\kappa}{4G}\int d^2x\sqrt{-g}\Big((\nabla\psi)^2+2R\psi-12(\nabla\psi)(\nabla\chi)\\ &- 12\psi\frac{(\nabla r)^2}{r^2}-12R\chi-12R\log r\Big)\Big],\end{aligned} \tag{9}$$

where, instead of the dilaton Φ we introduced the field $r = e^{-\Phi}$ which has the meaning of radius. $U(r) = 1 - \frac{Q^2}{r^2}$ is the dilaton potential. From the action (9) we get the following equations of motion:

$$\Box\psi = R \tag{10}$$

$$\Box\chi = \frac{(\nabla r)^2}{r^2} \tag{11}$$

$$2\Box r - rR - U' = -6\kappa\Big(2\psi\frac{\Box r}{r^2}+2\frac{(\nabla\psi)(\nabla r)}{r^2}-2\psi\frac{(\nabla r)^2}{r^3}+\frac{R}{r}\Big) \tag{12}$$

$$\begin{aligned}&g_{\mu\nu}(\Box r^2-(\nabla r)^2-U)-2r\nabla_\mu\nabla_\nu r = 2GT_{\mu\nu} =\\ &= \kappa\Big(g_{\mu\nu}(2R+6\psi\frac{(\nabla r)^2}{r^2}-\frac{1}{2}(\nabla\psi)^2+6(\nabla\psi)(\nabla\chi)-12\frac{\Box r}{r})\\ &+ \nabla_\mu\psi\nabla_\nu\psi-12\nabla_\mu\psi\nabla_\nu\chi-2\nabla_\mu\nabla_\nu\psi+12\nabla_\mu\nabla_\nu\chi\\ &+ 12\frac{\nabla_\mu\nabla_\nu r}{r}-12(1+\psi)\frac{\nabla_\mu r\nabla_\nu r}{r^2}\Big)\ .\end{aligned} \tag{13}$$

The static classical solution for the dilaton and metric is given by

$$r = x^1\ ,$$

$$g_{\mu\nu} = \begin{pmatrix} -f_0 & 0\\ 0 & \frac{1}{f_0}\end{pmatrix}\ ,$$

$$f_0 = \frac{(r-r_+)(r-r_-)}{r^2} = 1-\frac{2MG}{r}+\frac{Q^2}{r^2}\ .$$

This is the well known Reisner-Nordström solution which describes charged black hole of mass M and charge Q. The horizons of the black hole are $r_\pm = MG \pm \sqrt{(MG)^2-Q^2}$. The static solution for the auxilliary fields in the zero'th order is

$$\begin{aligned}\psi_0 &= Cr+2\log\frac{r}{l}+\frac{-r_++r_-+Cr_+{}^2}{r_+-r_-}\log\frac{r-r_+}{l}\\ &+ \frac{-r_++r_--Cr_-{}^2}{r_+-r_-}\log\frac{r-r_-}{l}\ ,\end{aligned} \tag{14}$$

$$\chi_0 = Dr - \frac{1}{3}\log\frac{r}{l} + \frac{-3r_+ + 6D{r_+}^2 + r_-}{6(r_+ - r_-)}\log\frac{r-r_+}{l} + \frac{-r_+ + 3r_- - 6D{r_-}^2}{6(r_+ - r_-)}\log\frac{r-r_-}{l} \ . \tag{15}$$

In order to have dimensionless expressions under the logarithm, we introduced one integration constant l of the dimension of length in both ψ_0 and χ_0. The constants C and D are introduced similarly as in the Schwarzchild case [8], and they are of great importance because they describe the quantum state of matter. We are interested to fix them in accordance with the Hartle-Hawking boundary conditions.

We want to calculate the one-loop quantum correction to the given classical solution. The static ansatz is the following:

$$r = x^1 \ , \tag{16}$$

$$g_{\mu\nu} = \begin{pmatrix} -fe^{2\phi} & 0 \\ 0 & \frac{1}{f} \end{pmatrix} \tag{17}$$

where

$$f = \frac{(r-r_+)(r-r_-)}{r^2} + \kappa\frac{m(r)}{r} \ ,$$

and

$$\phi(r) = \kappa\varphi(r) \ .$$

We can easily solve the equations of motion (12-13) in the first order in κ. The unknown functions $m(r)$ and $\varphi(r)$ in this order satisfy

$$m' = -\frac{2G}{\kappa}\frac{1}{f}T_{00} \tag{18}$$

$$\varphi' = \frac{2G}{\kappa}\frac{1}{2r}(T_{11} + \frac{T_{00}}{f^2}) \ . \tag{19}$$

The equations (18) and (19) can easily be integrated, but the solution for $m(r)$ and $\varphi(r)$ is so long that it makes no sense to write it in full length. We will first determine the values of the constants C and D using the strategy of [8]. Both m and φ are singular at the horizons r_+ and r_-: m has logarithmic singularities, while φ has logarithmic and $\frac{1}{r}$ singularities. Condition that the coefficients of all singular terms at the outer horizon r_+ vanish gives a relation between C and D:

$$6C{r_+}^3 + C(C-12D){r_+}^4 + 2r_+r_- - {r_-}^2 - {r_+}^2(1+2Cr_-) = 0 \ . \tag{20}$$

The other relation comes from the condition of regularity of the first correction of curvature at r_+,

$$R_1 = -\left(\frac{m}{r}\right)'' - {f_0}'\varphi' - 2f_0\varphi'' \ ,$$

and it is given by

$$-18C{r_+}^3 - C(C-12D){r_+}^4 - 26r_+r_- + 13{r_-}^2 + {r_+}^2(1+2Cr_-) = 0\ . \qquad (21)$$

The solution of (20) and (21) is

$$C = \frac{r_+ - r_-}{{r_+}^2}\ , \quad D = \frac{3r_+ - r_-}{6{r_+}^2}\ . \qquad (22)$$

Note that for $Q = 0$ ($r_- = 0$) the values of constants C and D are the same as in [8]; for $r_- = r_+$ we have $C = 0$.

Introducing the values (22) in the solution for m and φ, we get the quantum corrections of the metric tensor $g_{\mu\nu}$:

$$\begin{aligned} m(r) = & - \frac{1}{2{r_+}^4{r_-}^2r^3}\Big[-r^4(5{r_+}^2 - 6r_+r_- + {r_-}^2){r_-}^2 \\ & - r^2({r_+}^3 - 4{r_+}^2r_- + r_+{r_-}^2 - 6{r_-}^3){r_+}^2r_- \\ & - 2r(3{r_+}^2 + r_+r_- + 2{r_-}^2){r_+}^3{r_-}^2 + 4{r_+}^5{r_-}^3 \\ & + \log\frac{r}{r-r_-}\left(12r_+^4r_-^2r^2 - 6(r_+ + r_-)r_+^4r_-^2r + 4r_+^5r_-^3 + r_+^2(r_+^3 - 5r_+^2r_-)r^3\right) \\ & + \log\frac{r}{l}\left(12r_+^4r_-^2r^2 - 6(r_+ + r_-)r_+^4r_-^2r + 4r_+^5r_-^3 + r_+^2(r_-^3 - 5r_+r_-^2)r^3\right) \\ & - \log\frac{r-r_-}{l}\left((r_-^5 - 5r_+r_-^4)r^3 + 12r_+^2r_-^4r^2 - 6(r_+ + r_-)r_+^2r_-^4r + 4r_+^5r_-^3\right)\Big] \\ & + m_0 \end{aligned} \qquad (23)$$

and

$$\varphi(r) = F(r) - F(L)\ , \qquad (24)$$

where

$$\begin{aligned} F(r) = & - \frac{1}{2{r_-}^2{r_+}^4r^2(r-r_-)}\Big[2{r_+}^4{r_-}^3 + 2r(4r_+ - 3r_-){r_+}^2{r_-}^3 \\ & - r^2(3{r_+}^4 + 8{r_+}^3r_- - 4{r_+}^2{r_-}^2 - 4r_+{r_-}^3 + {r_-}^4)r_-\Big] \\ & + \log\frac{r}{r-r_-}\left(\frac{3}{r^2} + \frac{1}{r_+r_-} - \frac{3}{2r_-^2}\right) + \log\frac{r}{l}\left(\frac{3}{r^2} - \frac{3}{2r_+^2}\right) \\ & + \log\frac{r-r_-}{l}\left(\frac{3r_-}{r_+^3} - \frac{3r_-^2}{r_+^2r^2} - \frac{r_-^2}{2r_+^4} - \frac{1}{r_+^2}\right)\ . \end{aligned} \qquad (25)$$

This quantum correction, as we shall see, defines the Hartle-Hawking vacuum state. In the solution for m we entered arbitrary integration constant m_0. This constant is in principle related to the redefinition of the mass of black hole and we want to keep it to the end of calculations in order to check whether it influences the results. L is introduced to describe the size of our system; as in [15], we are considering black hole in thermodynamic equilibrium with radiation in the box of size L with ideally reflecting walls.

Let us rewrite the zero'th order solution for the auxilliary fields after the introduction of values (22) for C, D:

$$\psi_0 = \frac{r_+ - r_-}{{r_+}^2} r + 2\log\frac{r}{l} - \frac{{r_+}^2 + {r_-}^2}{{r_+}^2}\log\frac{r - r_-}{l} \ , \tag{26}$$

$$\chi_0 = \frac{3r_+ - r_-}{6{r_+}^2} r - \frac{1}{3}\log\frac{r}{l} - \frac{(r_+ - r_-)^2}{6{r_+}^2}\log\frac{r - r_-}{l} \ . \tag{27}$$

From the solutions (23-27) we see that the singularities on the inner horizon persisted, while singularities on the outer horizon vanished for the auxilliary fields, too.

If we define the null-coordinates for the Reisner-Nordström black hole as usual [16, 17]: $u = t - r_*$, $v = t + r_*$, where the "tortoise coordinate" r_* is given by

$$r_* = r + \frac{r_+^2}{r_+ - r_-}\log(r - r_+) - \frac{r_-^2}{r_+ - r_-}\log(r - r_-) \ , \tag{28}$$

the metric takes the form

$$ds^2 = -(1 - \frac{2M}{r} - \frac{Q^2}{r^2}) du dv. \tag{29}$$

The ingoing and outgoing fluxes are given by

$$\begin{aligned} T_{uu} &= T_{vv} = \frac{1}{48\pi}\frac{(r - r_+)^2}{4r^6{r_+}^4}\Big[8{r_+}^4{r_-}^2 - 4r(3{r_+}^2 + 2r_+r_- - 3{r_-}^2){r_+}^2 r_- \\ &+ 3r^2(r_+ - r_-)(r_+ + 7r_-){r_+}^2 - 2r^3(5{r_+}^2 - 6r_+r_- + {r_-}^2)r_+ \\ &- r^4(5{r_+}^2 - 6r_+r_- + {r_-}^2) \\ &+ 12{r_+}^2(r - r_-)^2\Big(({r_+}^2 + {r_-}^2)\log\frac{r - r_-}{l} - 2r_+^2\log\frac{r}{l}\Big)\Big] \ , \end{aligned} \tag{30}$$

while

$$T_{uv} = \frac{1}{48\pi r^6}(r - r_+)(r - r_-)(-3r_+r_- + 2rr_+ + 2rr_-) \ . \tag{31}$$

In order to investigate the regularity of EMT on the outer horizon, we have to use the free-falling observer frame [7]. The coordinates which describe this frame are the Kruscal $\{U, V\}$-coordinates. They are [17]

$$U = -e^{-\alpha u}, \quad V = e^{\alpha v} \ , \tag{32}$$

where

$$\alpha = \frac{r_+ - r_-}{2r_+^2} \ . \tag{33}$$

The components of EMT in the Kruscal coordinates take the form

$$T_{UU} = \frac{1}{\alpha^2 U^2} T_{uu} = \frac{V^2}{\alpha^2} e^{-4\alpha r}(r - r_-)^{\frac{2r_-^2}{r_+^2}} \frac{T_{uu}}{(r - r_+)^2} \tag{34}$$

$$T_{VV} = \frac{1}{\alpha^2 V^2} T_{vv} \tag{35}$$

$$T_{UV} = -\frac{1}{\alpha^2 UV} T_{uv} = \frac{1}{\alpha^2} e^{-2\alpha r} (r - r_-)^{\frac{r_-^2}{r_+^2}} \frac{T_{uv}}{r - r_+} \ . \tag{36}$$

So, from (30) and (31) we easily see that on the horizon $r = r_+$, i. e. $V = 0, U =$const, the regularity conditions

$$T_{UU} < \infty,\ T_{VV} < \infty,\ T_{UV} < \infty \tag{37}$$

or, equivalently,

$$\frac{T_{uu}}{f^2} < \infty,\ T_{vv} < \infty,\ \frac{T_{uv}}{f} < \infty \ , \tag{38}$$

are fulfilled for the values of constants C and D given by (22). These values of constants define that our sistem is in the thermal Hartle-Hawking state.

It is straightforward to confirm this statement also directly, along the lines of Balbinot, Fabbri, transforming the value of EMT from the Boulware state to the Hartle-Hawking conformal $|U, V\rangle$ state. Namely, in accordance with [20], the values of the EMT tensor in the null-coordinates in the Boulware state for the action (9) are given by

$$\begin{aligned} \left\langle B \right| \hat{T}_{uv} \left| B \right\rangle &= -\frac{1}{12\pi}(\partial_+\partial_-\rho + 3\partial_+\Phi\partial_-\Phi - 3\partial_+\partial_-\Phi) \\ &= \frac{1}{48\pi}(\frac{1}{2} f\, f'' + \frac{3}{r} f\, f') \ , \end{aligned} \tag{39}$$

and

$$\begin{aligned} \left\langle B \right| \hat{T}_{uu} \left| B \right\rangle &= \left\langle B \right| \hat{T}^{\rm PL}_{uu} \left| B \right\rangle + \frac{1}{2\pi}\Big(\rho(\partial_-\Phi)^2 + \frac{1}{2}\frac{\partial_-}{\partial_+}(\partial_+\Phi\partial_-\Phi)\Big) \\ &- \frac{1}{4\pi}(-2(\partial_-\rho)(\partial_-\Phi) + (\partial_-\Phi)^2) \\ &= \left\langle B \right| \hat{T}^{\rm PL}_{uu} \left| B \right\rangle + \frac{1}{16\pi}\frac{f^2}{r^2}\log f, \end{aligned} \tag{40}$$

where $\rho = \frac{1}{2}\log f$ is the conformal factor and $f = f_0 = \frac{(r-r_-)(r-r_+)}{r^2}$. $\left\langle B \right| \hat{T}^{\rm PL}_{uu} \left| B \right\rangle$ is the value of EMT for the case where the complete effective action consists of the Polyakov-Liouville term only. The conformal transformation to the Hartle-Hawking state $|U, V\rangle$ gives for the value of EMT

$$\left\langle H \right| \hat{T}_{uv} \left| H \right\rangle = \left\langle B \right| \hat{T}_{uv} \left| B \right\rangle \tag{41}$$

$$\begin{aligned} \left\langle H \right| \hat{T}_{uu} \left| H \right\rangle &= \left\langle B \right| \hat{T}_{uu} \left| B \right\rangle + \frac{1}{24\pi}(\frac{F''}{F} - \frac{1}{2}\frac{F'^2}{F^2}) \\ &+ \frac{1}{4\pi}\Big((\partial_-\Phi)^2 \log FG + \frac{F'}{F}\int(\partial_+\Phi)(\partial_-\Phi)dv\Big) \ . \end{aligned} \tag{42}$$

Here, $F(u) = \frac{du}{dU}$, $G(v) = \frac{dv}{dV}$. It if easy to check that, when calculated, (42) really coincides with the value (30) obtained previously.

Knowledge of the functions m and φ allows us to calculate the first quantum corrections of the positions of the horizons, temperature and entropy of the charged black hole. The values (22) assure regularity of these corrections, as we shall shortly see. If we define the corrected positions of inner and outer horizons as

$$r_+^q = r_+ + \kappa r_1 \ , \quad r_-^q = r_- + \kappa r_2 \ ,$$

we get $r_1 = -\frac{r_+}{r_+ - r_-} m(r_+)$, $r_2 = -\frac{r_-}{r_- - r_+} m(r_-)$. Using (23), we obtain

$$\begin{aligned} r_1 \quad &= \quad -\frac{1}{2{r_+}^3{r_-}^2}\Big[r_+r_-({r_+}^2 + 8r_+r_- + {r_-}^2) - (r_+^4 + 2r_+^2r_-^2 - 4r_+^3r_-)\log\frac{r_+}{r_+ - r_-} \\ &- \quad {r_+}^2{r_-}^2\log\frac{r_+}{l} + {r_-}^4\log\frac{r_+ - r_-}{l}\Big] - \frac{r_+}{r_+ - r_-}m_0 \end{aligned} \qquad (43)$$

for the correction of the outer horizon.

The Hawking temperature is defined by

$$T = \frac{1}{4\pi} e^{\phi} f'|_{r_+^q} = T_0 + \kappa T_1$$

where, as it is known, $T_0 = \frac{1}{4\pi}\frac{r_+ - r_-}{r_+^2}$. The first correction of temperature T_1 in the case of nonextreme Reisner-Nordström solution is given by

$$\begin{aligned} 4\pi T_1 \quad &= \quad \frac{r_+ - r_-}{{r_+}^2}\phi(r_+) + \frac{m'(r_+)}{r_+} + \frac{m(r_+)(r_+ - 3r_-)}{{r_+}^2(r_+ - r_-)} = \\ &= \quad \frac{1}{2{r_+}^6 r_-}\Big(4{r_+}^4 + 5{r_+}^3r_- - 33{r_+}^2{r_-}^2 - r_+{r_-}^3 + {r_-}^4\Big) \\ &+ \quad (\frac{4}{r_+^3} + \frac{2}{r_+r_-^2} - \frac{6}{r_+^2r_-})\log\frac{r_+ - r_-}{r_+} \\ &+ \quad (\frac{4r_-}{r_+^4} - 6\frac{r_-^2}{r_+^5} + 2\frac{r_+^3}{r_-^6})\log\frac{r_+ - r_-}{l} \\ &- \quad \frac{r_+ - r_-}{r_+^2}F(L) + \frac{m_0(r_+ - 3r_-)}{r_+^2(r_+ - r_-)} \ . \end{aligned} \qquad (44)$$

The standard definition of entropy, based, e.g., on the conical singularity method [8]:

$$S = \frac{\pi}{G}(r^2 - \kappa(2\psi - 12\chi - 12\log r))|_{r_+^q} = S_0 + \kappa S_1 \ , \qquad (45)$$

gives for its correction linear in κ

$$\begin{aligned} S_1 \quad &= \quad \frac{\pi}{G}\Big[\frac{2m_0}{r_+(r_+ - r_-)} - 4 - \frac{r_+}{r_-} - \frac{r_-}{r_+} + 5\log\frac{r_+}{l} \\ &+ \quad (2 + \frac{r_+^2}{r_-^2} - \frac{4r_+}{r_-})\log\frac{r_+}{r_+ - r_-} + (\frac{4r_-}{r_+} - \frac{r_-^2}{r_+^2})\log\frac{r_+ - r_-}{l}\Big], \end{aligned} \qquad (46)$$

up to constants. This represents the quantum correction to the standard Bekenstein-Hawking entropy.

3 Extreme black hole

Extreme Reisner-Nordström black hole is defined by the coincidence of the inner and outer horizons, $r_- = r_+$. In this case the function $f(r)$ which defines the metric has a double zero at $r = r_+ = MG$. Notice that, technically, this implies difference in the calculations which were performed, e.g. the types of singularities which we obtained are different.

Let us assume first that the extreme black hole can be approached in the limit of nonextreme ones, e.g. in a way similar to that of Zaslavskii [18, 19]. We shall assume that this means taking the limit $r_- \to r_+$ in the results that are already obtained for the nonextreme solutions, i.e. at the end of calculations. Let us notice that this fixes the value of the constant C in the auxilliary field ψ to zero: $C = \frac{r_+ - r_-}{{r_+}^2} = 0$, while for D we get $D = \frac{1}{3r_+}$. The values of C and D give the change of the metric at infinity, telling us what is the temperature (energy) of the Hawking radiation. $C = 0$ corresponds to the case that the metric remains Minkowskian in the asymptotic region, i.e. that the temperature of radiation is zero. So, extreme limit taken in this sense really gives zero temperature.

The functions m, φ and F which we have calculated in this case reduce to:

$$m(r) = \frac{2(r - r_+)}{r^3 r_+}\Big[r_+(r_+ - 2r) + 2(r - r_+)^2 \log\frac{r}{r - r_+}\Big] + m_0 \tag{47}$$

$$F(r) = \frac{1}{r^2 {r_+}^2}\Big[r_+(2r + r_+) - 2(r^2 - 3{r_+}^2)\log\frac{r}{r - r_+}\Big]\ . \tag{48}$$

The first quantum correction of the curvature is

$$R_1(r) = -\frac{4}{r^6 r_+}\Big[r_+(-2r^2 + 19 r r_+ - 19{r_+}^2) + 2(r^3 - 9r{r_+}^2 + 8{r_+}^3)\log\frac{r}{r - r_+}\Big]\ . \tag{49}$$

From (47-49) we see that m and R_1 are not singular at the horizon, while φ is.

The corrections of the position of the horizon which we get in this case are

$$r_1 = -\frac{5}{r_+}\ , r_2 = \frac{5}{r_+}\ . \tag{50}$$

if we assume that the constant $m_0 = 0$; otherwise the result is divergent. From the last expression we can see that classical extreme black hole would turn to the nonextreme when quantum corections are taken, i. e. $r_+^q \neq r_-^q$.

For the correction of the temperature we get

$$4\pi T_1 = -\frac{12}{{r_+}^3}\ , \tag{51}$$

for $m_0 = 0$. This would mean that the quantum corrected temperature is negative, because, as we have already seen, the classical temperature is zero, $T_0 = 0$. This result is not physically acceptable. The one-loop correction of the entropy

$$S_1 = \frac{\pi}{G}[\,-6 + 4\log\frac{r_+(r_+ - r_-)}{l^2}]|_{r_- = r_+}$$

is divergent and therefore also difficult to interpret.

To conclude the discussion of this case, in order to compare our results with [5, 6], let us analyze the behavior of the EMT. The nonsingularity of EMT in the free-falling observer frame [6] gives that T_{vv}, T_{uv}/f and T_{uu}/f^2 are nonsingular on the horizon. For T_{uu} we get

$$T_{uu} = T_{vv} = -\frac{(r-r_+)^3}{24\pi r^6}(r_+ + 3(r-r_+)\log\frac{r}{r-r_+}) \tag{52}$$

while

$$T_{uv} = \frac{r_+(r-r_+)^2(4r-3r_+)}{48\pi r^6} \ . \tag{53}$$

Since the metric factor is $f = \frac{(r-r_+)^2}{r^2}$, we see that the EMT is divergent at the horizon unlike the nonextreme case, as Trivedi also notices [5]. This result implies that the state defined by the choice $C = 0, D = \frac{1}{3r_+}$ is not thermal. Let us also note that the values of EMT (52), (53) coincide with the values of EMT for the Boulware state, $\left\langle B \middle| \hat{T}_{uu} \middle| B \right\rangle$ and $\left\langle B \middle| \hat{T}_{uv} \middle| B \right\rangle$, calculated in accordance to [20].

Let us finally note that, as our results also show, the "extreme limit" in the sense which we have just discussed could not exist in reality, i.e. in physical experiment. This is because the temperature of nonextreme black holes decreases towards zero in the limit $MG \to Q$, so in thermodynamic sense approaching the extreme black hole would be the same as approaching the absolute zero.

Now we turn to the other possibility, namely finding the quantum corrections of the extreme black hole solution from the beginning. This procedure is justified by the fact that the topology of extreme solution is different from the topology of nonextreme one [4]. The ansatz for the correction is

$$r = x^1 \ ,$$

$$g_{\mu\nu} = \begin{pmatrix} -fe^{-2\Phi} & 0 \\ 0 & \frac{1}{f} \end{pmatrix}$$

where now the function f is given by

$$f = \frac{(r-r_+)^2}{r^2} + \kappa\frac{m(r)}{r} \ .$$

The zero'th order solutions for the auxilliary fields are

$$\psi_0 = Cr - \frac{C{r_+}^2}{r-r_+} + 2\log\frac{r}{l} + 2Cr_+\log\frac{r-r_+}{l} \ , \tag{54}$$

$$\chi_0 = Dr - \frac{1}{3}\log\frac{r}{l} + \frac{r_+(1-3Dr_+)}{3(r-r_+)} - \frac{2(1-3Dr_+)}{3}\log\frac{r-r_+}{l} \ . \tag{55}$$

Note that (54) and (55) are not the same as (14) and (15) in the limit $r_+ \to r_-$. Proceeding on the similar lines as in the previous section, we can solve the equations for m and φ and impose the conditions of regularity. It is interesting that now also a choice of constants C and D exists which enables the regularity m, φ, R_1 on the horizon, and EMT in the free falling observer frame. It is given by

$$C = \frac{1}{r_+} \; , D = \frac{5}{12 r_+} \; . \tag{56}$$

For m and φ in this case we get

$$m(r) = \frac{2}{{r_+}^2 r^3} \Big[r^4 + 4{r_+}^3 r - 2{r_+}^4 + 2r_+(r - r_+)^3 \log\frac{r}{l} \Big] + m_0 \tag{57}$$

$$\varphi(r) = F(r) - F(L) \; , \tag{58}$$

where

$$F(r) = \frac{2}{{r_+}^2 r^2} \Big[2r_+(2r + r_+) - (r^2 - 3{r_+}^2) \log\frac{r}{l} \Big] \; . \tag{59}$$

The auxilliary fields become

$$\psi(r) = \frac{r}{r_+} - \frac{r_+}{r - r_+} + 2\log\frac{r}{l} \; , \tag{60}$$

and

$$\chi(r) = \frac{1}{12} \Big[\frac{5r}{r_+} - \frac{r_+}{r - r_+} - 4\log\frac{r}{l} + 2\log\frac{r - r_+}{l} \Big] \; . \tag{61}$$

We see that for the values (56) the auxilliary fields diverge at $r = r_+$, unlike m and φ.

It is easy to see that the equation defining the position of the horizon r_+^q is not compatible with the ansatz $r_+^q = MG + \kappa r_1$. If we take the ansatz in the form $r^q = M + \sqrt{\kappa} r_1$, we get

$$r_1^2 = -6 - M m_0 \; . \tag{62}$$

We see that, although this expression can be positive (for $m_0 < -\frac{6}{M}$), it is physically rather unacceptable to have the correction r_1 dependent on the arbitrary parameter m_0. This indicates that the correction of the horizon is probably nonanalytic.

The components of EMT for this solution are

$$T_{uu} = T_{vv} = -\frac{(r - r_+)^4}{48\pi r^6 {r_+}^2} (r^2 + 4 r r_+ + {r_+}^2 + 6{r_+}^2 \log\frac{r}{l}) \; , \tag{63}$$

$$T_{uv} = \frac{r_+ (r - r_+)^2 (4r - 3r_+)}{48\pi r^6} \; . \tag{64}$$

The asymptotic value of energy density T_{00} is $-\frac{1}{48\pi {r_+}^2}$ and does not vanish, which means that the temperature of the radiation is probably not zero. Further, the factor $(r - r_+)^4$ in (63) ensures the regular behaviour of all components of EMT in the free

falling observer frame at the horizon, in accordance with the condition (38). Note that in [5], where the effective action consists of the Polyakov-Liouville term only, the choice of constants C and D which ensures the regular behaviour does not exist. This is a particular property of the SSG model.

In order to calculate entropy, let us analyze the properties of the Euclidean extension of the extreme Reisner-Nordström metric. Near the horizon, r_+^q, the Euclidean metric $ds^2 = fe^{2\phi}d\tau^2 + \frac{1}{f}dr^2$, where $\tau \in [0, 2\pi\bar{\beta}]$ $(2\pi\bar{\beta} = \bar{T}^{-1})$, does not posess the conical singularity and this means that the temperature of the black hole is arbitrary [3]. In order to find the corresponding entropy, let us write the Euclidean effective action with the appropriate surface terms added (see Appendix of [8]):

$$\begin{aligned}
\Gamma_1 &= -\frac{1}{4G}\Big(\int_{\tilde{M}_\alpha} d^2x\sqrt{g}[r^2R + 2(\nabla r)^2 + 2U(r)] \\
&- \kappa\int d^2x\sqrt{g}[2R(\psi - 6\chi) + (\nabla\psi)^2 \\
&- 12(\nabla\psi)(\nabla\chi) - 12\frac{\psi(\nabla r)^2}{r^2} - 12R\log r]\Big) \\
&- \frac{1}{2G}\Big(\int_{\partial\tilde{M}_\alpha} r^2kds - \kappa\int_{\partial\tilde{M}_\alpha}(2\psi - 12\chi - 12\log r)kds\Big)\ . \qquad (65)
\end{aligned}$$

After a simple calculation, using the expression for external curvature of the boundary of the manifold, $k = \frac{f'}{2f} + \phi'\sqrt{f}$, we get

$$\begin{aligned}
\Gamma_1 &= \frac{\pi\bar{\beta}}{G}\int_{r_+}^{L} dx[2rr'(\sqrt{f}e^{\phi})'\sqrt{f} - fr'^2 - 2U(r)] \\
&+ \frac{\kappa\pi\bar{\beta}}{G}\int_{x_+}^{L} dx\Big((\sqrt{f}e^{\phi})'(2\psi - 12\chi - 12\log r)' \\
&+ \frac{1}{2}f\psi'^2 - 6f\psi'\chi' - 6\frac{\psi fr'^2}{r^2}\Big) \qquad (66)
\end{aligned}$$

From (66) we see that Γ_1 is proportional to $\bar{\beta}$, which means that the entropy is equal to zero. The zero entropy is connected to the fact that the proper distance between the horizon $r = r_+$ and any other point of the manifold is infinite: extreme black hole has zero entropy and arbitrary temperature. On the other hand it seems that temperature of the scalar field gas is not arbitrary and this can be a consequence of the fact that the heat capacity of the extreme black hole is zero [21].

4 Conclusions

We calculated the quantum corrections for the Reisner-Nordström charged black hole. The backreaction of the radiation is described by the SSG effective action (9), in which the effects of nonlocality are expressed through the auxilliary fields ψ and χ.

In the case of the nonextreme black hole the semiclassical procedure for finding the correction is unambiguously defined and can be performed similarly to the

Schwarzchild case. The Hartle-Hawking vacuum state which describes the thermalized black hole in equilibrium with radiation is defined by choosing the values of constants C and D. Here, also, the asymptotic value of the energy density is negative, which is a characteristic of the SSG model as it was discussed in [8, 20]. The analysis of the extreme Reisner-Nordström case turns out to be more difficult. First, treatment along the lines of Zaslavskii gives some physically unacceptable results. This is related with the fact that the extreme black holes are topologically different from the nonextreme ones. In the framework of the Hawking approach, the behaviour of the Euclidean extension shows that the temperature of the extreme Reisner-Nordström black hole is arbitrary, while its entropy is zero. On the other hand, our analysis of the EMT of radiation (in the SSG model) shows that there exists some value of temperature for which the energy-momentum tensor is regular everywhere. But, at the same time, we see that the correction of the position of the horizon is nonperturbative. This is probably the consequence of the fact that the horizon of the extreme black hole is not bifurcate. The coordinates which are regular at the horizon are not of the Kruscal type $U = e^{-\alpha u}$, $V = e^{\alpha v}$ [22], and the horizon consists of the four unconnected branches. This might be the physical reason for the impossibility of defining of the Hartle-Hawking vacuum by choosing special coordinates.

Appendix

In this appendix we will analyze the general model where the conformal term $\xi f_i^2 R$ is included in 4D action, i.e the action given by (1). We want to calculate the one-loop effective action taking the quantum correction for the matter field f_i only. In short, we will repeat the steps in [9, 14], for this model. If we rescale the field of matter, $f_i \to e^{\Phi} f_i$ we get

$$\begin{aligned}\Gamma_m &= -\frac{1}{2}\sum_i \int d^2x\sqrt{-g}((\nabla f_i)^2 + 2f_i\nabla f_i\nabla\Phi + f_i^2(\nabla\Phi)^2) \\ &- \frac{\xi}{2}\sum_i \int d^2x\sqrt{-g} f_i^2(R + 4\Box\Phi - 6(\nabla\Phi)^2 + 2e^{2\Phi}) \end{aligned} \qquad (A.1)$$

Using the complexification of the fields (the doubling trick) we get

$$\begin{aligned}\Gamma_m &= -\sum_i \int d^2x\sqrt{-g}(\nabla f_i^*\nabla f_i - \Box\Phi f_i^* f_i + (\nabla\Phi)^2 f_i^* f_i) \\ &- \xi\sum_i \int d^2x\sqrt{-g} f_i^* f_i(R - 6(\nabla\Phi)^2 + 4\Box\Phi + 2e^{2\Phi}) \ . \end{aligned} \qquad (A.2)$$

Expanding the background metric around a flat metric, $g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}$ the action (A.2) becomes

$$\begin{aligned}\Gamma_m &= \sum_i \int d^{2+\epsilon}x f_i^*\Big[\eta^{\mu\nu}\partial_\mu\partial_\nu + \overleftarrow{\partial_\mu}\bar{\gamma}^{\mu\nu}\overrightarrow{\partial}_\nu \\ &+ \sqrt{-g}(\Box\Phi - (\nabla\Phi)^2 - \xi(R - 6(\nabla\Phi)^2 + 4\Box\Phi + 2e^{2\Phi}))\Big] f_i \ , \end{aligned} \qquad (A.3)$$

where $\bar{\gamma}^{\mu\nu} = \gamma^{\mu\nu} - \frac{1}{2}\gamma\eta^{\mu\nu}$. The vertices with two and zero spacetime derivatives we denote as A and C (like as in [9, 14]). The corresponding diagrams are given by

$$C = -\frac{iN\pi^{\frac{d}{2}}}{(2\pi)^2}\Gamma(-\frac{\epsilon}{2})\int d^2x\sqrt{-g}(-\xi R - 2\xi e^{2\Phi} - (1-6\xi)(\nabla\Phi)^2 + (1-4\xi)\Box\Phi)$$

$$AC = -\frac{iN\pi^{\frac{D}{2}}}{(2\pi)^2}\Big(\int d^2x\sqrt{-g}R\frac{1}{\Box}\big[-\xi R - 2\xi e^{2\Phi} - (1-6\xi)(\nabla\Phi)^2\big] + (1-4\xi)\int d^2x\sqrt{-g}R\Phi\Big)$$

$$AA = -\frac{iN\pi^{\frac{D}{2}}}{(2\pi)^2}\Gamma(1-\frac{\epsilon}{2})B(2+\frac{\epsilon}{2}, 2+\frac{\epsilon}{2})\int d^2x\sqrt{-g}\Big(R\frac{1}{\Box}R + \frac{4}{\epsilon(1+\frac{\epsilon}{2})}R\Big)\ .$$

The one-loop correction to the effective action is given by [9, 14]

$$\Gamma^{(1)} = \frac{i}{2}(C - AC - \frac{1}{2}AA)$$

Using (A.2) we get the effective action (3). Given this action we can investigate the vacuum solutions of the corresponding equations of motion along the lines developed above and in [8]. Unfortunately, this does not change our results essentialy. The flux of radiation at large radius remains negative, $T_{uu} = \frac{(r_+ - r_-)(-5r_+ + r_-)}{192\pi r_+^3}$, which was one point where one might expect improvement. In the analysis of the extreme solution one can see that the regular behavior of EMT exists only in the case $\xi = 0$, which is presented in detail.

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